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This booklet introduces high school students to math applications used in flight. There are 4 units, each with examples and exercises. Each unit begins with general math concepts, progressing to higher level math concepts.
The Wright brothers used many calculations to create their flying machine. The following formulas are examples of some calculations commonly performed by flight personnel today.

\[ \text{distance} = \text{rate} \times \text{time} \]

or \[ d = r \times t \]

This formula is used to find the distance a plane will fly on a trip. It can also be used to find the speed of the plane when the distance and time are given, or to find the time when the distance and speed are given.

Distance was one measure of success for the Wright brothers. Distance usually is measured in miles. Rate, or speed, usually is measured in knots, or nautical miles per hour. Nautical miles are greater than ground miles due to the earth’s curvature. A nautical mile is 6080.27 feet. A ground mile is 5280 feet. Time usually is measured in hours.

**Example:**

A plane travels at 350 knots (nautical miles per hour) for 3 hours. How many nautical miles will the plane travel?

**Solution:**

\[ d = r \times t \]

\[ d = 350 \times 3 \]

\[ d = 1,050 \]

Since rate and time both have been measured in hours, multiplication is all that is necessary.

The plane will travel 1,050 nautical miles.

**Exercise 1:**

A plane travels 500 knots for 4 hours. How many nautical miles will the plane travel?

**Exercise 2:**

A plane travels 2,550 nautical miles in a 5-hour trip. What is the speed of the plane?
The Wright brothers used a canard to control the front of the plane. A canard is a flap that directs the wind up or down so that the plane does not dive or climb out of control. The canard affected the center of gravity, the point where the weight of the plane is concentrated. Modern engineering proved that a canard could be replaced by similar flaps on the wings, moving the plane’s center of gravity toward the middle of the plane. This improved flight performance. The center of gravity still is calculated by flight engineers today. The formula is:

$$c_g = \frac{\text{Total moments}}{\text{Total weight}}$$

where c.g. is the center of gravity. This is important so that the plane is balanced side-to-side and front-to-back. Balancing the plane puts equal force on each wing and equalizes the force between the nose and the tail of the plane.

A moment is force an object creates based on its weight and position in the plane. A heavy item placed farther from the center of gravity will create a higher moment.

**Example:**

A pilot has added the weights and moments for the following: Plane weight when empty, Weight of pilot and passengers, Baggage, Oil and Fuel. The total weight of the items is 1,317.5 pounds. The total of their moments is 108200. Find the center of gravity. Round answer to the nearest tenth.

**Solution:**

$$c_g = \frac{\text{Total moments}}{\text{Total weight}}$$

$$c_g = \frac{108,200}{1,317.5}$$

$$c_g = 82.1$$

The pilot then refers to a graph called the Center of Gravity Moment Envelope to see that 82.1 falls within the Normal category for the calculated weight and moment.

**Exercise 1:**

Calculate the center of gravity for a total weight of 2275 and a total moment of 100,000.

**Exercise 2:**

Calculate the total moment given a center of gravity of 78.5 and a total weight of 8,000.
Atmospheric Pressure

The early flights of the Wright brothers were typically 10 to 12 feet above the ground. It was five years later, in 1908, before their flying machine could reach an altitude of 100 feet. Today commercial jets fly 30,000 to 40,000 feet. At such high altitudes, the atmospheric pressure is much different. Flight personnel calculate the atmospheric pressure using the following formula, where \( a \) is the altitude of the plane, measured in feet, and \( p \) is the atmospheric pressure, measured in pounds per square inch.

\[
p = \frac{-9.05 \left[ \left( \frac{a}{1,000} \right)^2 - \frac{65a}{1,000} \right]}{\left( \frac{a}{1,000} \right)^2 + 40 \left( \frac{a}{1,000} \right)}
\]

**Example:**

Calculate the atmospheric pressure for a plane flying at 10,000 feet. Round answer to the nearest tenth.

**Solution:**

\[
p = \frac{-9.05 \left[ \left( \frac{a}{1,000} \right)^2 - \frac{65a}{1,000} \right]}{\left( \frac{a}{1,000} \right)^2 + 40 \left( \frac{a}{1,000} \right)}
\]

\[
p = \frac{-9.05 \left[ \left( \frac{10,000}{1,000} \right)^2 - \frac{65(10,000)}{1,000} \right]}{\left( \frac{10,000}{1,000} \right)^2 + 40 \left( \frac{10,000}{1,000} \right)}
\]

\[
p = \frac{-9.05 \left[ (10)^2 - 65(10) \right]}{(10)^2 + 40(10)}
\]

\[
p = \frac{-9.05 [100 - 650]}{100 + 400}
\]

\[
p = \frac{-9.05 [-550]}{500}
\]

\[
p = \frac{4,977.5}{500}
\]

\[
p = 9.955 = 10.0
\]

**Exercise 1:**

Calculate the atmospheric pressure for a plane flying at 5,000 feet.

**Exercise 2:**

Calculate the atmospheric pressure for a plane flying at 20,000 feet.

Round answers to the nearest tenth.

The atmospheric pressure is 10.0 pounds per square inch.
The first successful flight of the Wright brothers lasted 59 seconds covering 852 feet. Today airplanes fly for hours across the country and around the world.

The continental United States is divided into four **time zones**. When it is 10:00 a.m. in the Eastern time zone, it is only 7:00 a.m. in the Pacific time zone.

In aviation, time is represented without the *a.m.* or *p.m.* Time is given with four digits. The left two indicate the hour, the right two indicate the minutes. 7:00 a.m. would be 0700. 10:00 a.m. would be 1000. Since there is also 7:00 p.m., time is measured on a 24-hour basis. 7:00 p.m. would be 12 hours past 7:00 a.m., so 7:00 p.m. would be 0700 + 12 hours. The 12 would be added to the hours digits, giving 1900. This would be read nineteen hundred hours. 0700 would be read O seven hundred hours. This format is most commonly used in the military and is therefore called military time.

**Example:**

An airplane leaves an airport in the Central standard time zone at 0730 CST for a 2-hour flight to a destination in the Mountain standard time zone. What time is the airplane expected to land?

**Solution:**

0730 + 2 hours = 0930 CST

Mountain time is one hour earlier than Central time, so subtract one hour to convert 0930 CST to Mountain standard time.

0930 – 1 hour = 0830 MST

The airplane is expected to land at 8:30 a.m. MST.
Time Zones 2

All time zones around the world are based on Greenwich (Gren-itch) Mean Time, the time in Greenwich, England. Greenwich Mean Time, referred to as Zulu time, is five hours ahead of the Eastern time zone.

Example:

Give the Zulu time for 0945 EST.

Solution:

Add 5 to the hours digits (left 2) of the Eastern time.

$$0945 + 5 \text{ hours} = 1445 \text{ Z}$$

The Zulu time is 1445 Z.

Exercise 1:

Give the Zulu time when it is 4:30 p.m. MST.

Exercise 2:

Give the Zulu time when it is 8:15 p.m. CST.
Wilbur and Orville Wright were constantly correcting their flying machine’s direction, or heading, during their flights. They were able to do this by pulling cables attached to flaps and flexible wings.

Direction or heading is measured in degrees starting from due north, progressing clockwise. Pilots use an instrument called the heading indicator to monitor the plane’s direction. To calculate the heading, multiply the heading indicator reading by 10.

Give the headings shown by these heading indicators.

**Examples:**

![Heading Indicator A](image1)

A. 3 x 10 = 30º

![Heading Indicator B](image2)

B. 21 x 10 = 210º

![Heading Indicator C](image3)

C. 27 x 10 = 270º

**Solutions:**

A. 3 x 10 = 30º  
B. 21 x 10 = 210º  
C. 27 x 10 = 270º

**Exercise 1:**

Complete the heading indicator to show a heading of 60º.

![Exercise 1](image4)

**Exercise 2:**

Complete the heading indicator to show a heading of 190º.

![Exercise 2](image5)
The first runway was a sandy beach in Kitty Hawk, North Carolina. The Wright brothers chose this for soft landings, no obstacles and plenty of wind. Today airports have multiple runways. Each is clearly marked with its heading.

**Example:**

Runway 18 indicates a heading of 180º.  
(18 x 10 = 180)

Runway 36 indicates a heading of 360º.  
(36 x 10 = 360)

(Note: A heading of 360º is the same as 0º, but airports do not designate any runways as Runway 0.)

Wind direction determines which runway is used for take-off and landing. Planes fly into the wind to take off and land.

**Exercise 1:**

Using a protractor, draw and label a runway with a heading of 50º. Label the opposite direction also.

**Exercise 2:**

Using a protractor, give the headings for both directions for this runway.
Atmospheric Pressure and Altitude

The early flights of the Wright brothers were typically 10 to 12 feet above the ground. It was 5 years later, in 1908, before their flying machine could reach an altitude of 100 feet. Today commercial jets fly 30,000 to 40,000 feet. At such high altitudes, the atmospheric pressure is much different.

Atmospheric pressure decreases approximately 1 inch of mercury (Hg) per 1000 feet of altitude gained. Lowering the altimeter setting lowers the altitude reading. If a pilot makes the following changes to the altimeter setting, what is the approximate change in altitude?

Example: Setting changed from 30.18” to 29.86”.

Solution:

\[
\frac{1”}{1000’} = \frac{-0.32”}{X}
\]

\[X = -320 \text{ feet lower}\]

Exercise 1: Setting changed from 29.25” to 29.95”.

Exercise 2: Setting changed from 30.30” to 29.41”.

Exercise 3: Setting changed from 30.00” to 29.67”.
The rate at which a plane descends is referred to as the **slope of descent**. It is defined the same as the slope is in graphing:

\[
\text{Slope} = \frac{\text{Change in } y \text{ (vert. direction)}}{\text{Change in } x \text{ (horiz. direction)}} = \frac{\text{rise}}{\text{run}}
\]

The slope of descent often is given as a percent.

**Example:**

Find the approximate slope of descent, expressed as a percent, if a plane is flying at 31,300 feet, intending to land 60 miles away. The elevation of the landing site is 2500 feet.

**Solution:**

Using a diagram, we can determine

\[
\text{Slope} = \frac{\text{rise (or descent)}}{\text{run}}
\]

The plane will descend

\[
31300 \text{ feet} - 2500 \text{ feet} = 28800 \text{ feet}
\]

over 60 miles.

Ratios compare like units. To create the ratio of the slope, 60 miles must be converted to feet.

\[
60 \text{ miles} \times \frac{5280 \text{ feet}}{1 \text{ mile}} = 316800 \text{ feet}
\]

The ratio for the slope is

\[
\frac{28800 \text{ feet}}{316800 \text{ feet}} = \frac{1}{11}
\]

This means for every 1 foot the plane descends, it covers 11 feet, or for every 1 mile the plane descends, it covers 11 miles. Since a ratio is a comparison of like units, we can use feet or miles to describe the ratio.

The slope of descent is often given as a percent. To convert \(\frac{1}{11}\) to a percent, convert to a decimal by dividing, then move the decimal point 2 places to the right.

\[
\frac{0.0909}{11} = \frac{0.91}{100} = 9.1\%
\]

**Exercise 1:**

Find the approximate slope of descent, expressed as a percent, if a plane is flying at 23800 feet, intending to land 50 miles away. The elevation of the landing site is 1000 feet.

**Exercise 2:**

Find the distance in miles needed to land a plane flying at 40000 feet if the landing site is at 2000 feet elevation and the slope of descent is 20%.
The Wright brothers needed strong winds to create lift for the wings of their flying machine. They experimented in winds as strong as 30 miles per hour. To study the effect of wind on the wings of an airplane, they built a wind tunnel. It was 6 feet long and 16 inches square. A small engine similar to our lawn mower engines powered a fan at one end to create various wind speeds.

A **tailwind** is wind moving in the same direction as the airplane. This pushes the plane, increasing the airplane’s speed.

A **headwind** is wind moving in the opposite direction of the airplane. This slows the airplane’s speed.

Airplane speed is commonly measured in knots, or nautical miles per hour. Due to the curve of the earth’s surface, a nautical mile (6080.27 feet) is greater than a ground mile (5280 feet).

### Example 1:
A plane’s speed in still air is 275 knots. The speed of a headwind is 25 knots. Calculate the speed of the plane traveling into the headwind.

**Solution:**
To find the speed of a plane traveling into a headwind, subtract the headwind speed from the plane’s speed in still air.

\[
275 - 25 = 250
\]

The plane is traveling at 250 knots.

### Example 2:
A plane’s speed in still air is 275 knots. The speed of a tailwind is 25 knots. Calculate the speed of the plane traveling with the tailwind.

**Solution:**
To find the speed of a plane traveling with a tailwind, add the tailwind speed to the plane’s speed in still air.

\[
275 + 25 = 300
\]

The plane is traveling at 300 knots.

### Exercise 1:
A plane’s speed in still air is 400 knots. The speed of a headwind is 35 knots. Calculate the speed of the plane traveling into the headwind.

### Exercise 2:
A plane’s speed in still air is 210 knots. The speed of a tailwind is 15 knots. Calculate the speed of the plane traveling with the tailwind.
Headwinds and Tailwinds 2

Example:

With a tailwind, a plane travels 210 miles in 45 minutes. With a headwind, it travels 210 miles in one hour. Find the speed of the plane in still air.

Solution:

Since knots are nautical miles per hour, we should change the minutes to hours.

\[
\text{45 minutes} \times \frac{1 \text{ hour}}{60 \text{ minutes}} = \frac{45}{60} = \frac{3}{4} \text{ hour}
\]

In the chart above, \( p \) represents the speed of the plane in still air; \( w \) represents the speed of the wind.

(Note: Because the distance in both situations (tailwind and headwind) is 210 miles, the two equations for distance can be set equal.)

\[
\frac{3}{4}(p + w) = p - w
\]

\[
4\left[\frac{3}{4}(p + w)\right] = 4(p - w) \quad \text{to clear the fraction}
\]

\[
3(p + w) = 4(p - w)
\]

\[
3p + 3w = 4p - 4w
\]

\[
3w = 4p - 4w - 3p
\]

\[
3w = p - 4w
\]

\[
3w + 4w = p
\]

\[
7w = p
\]

The plane traveled 210 nautical miles with the tailwind and 210 nautical miles with the headwind. Use either distance from the chart equal to 210, substituting 7w for \( p \) to find \( w \), the speed of the wind:

\[
p - w = 210
\]

\[
7w - w = 210
\]

\[
6w = 210
\]

\[
\frac{1}{6}6w = 210\left(\frac{1}{6}\right)
\]

\[
w = 35
\]

The wind speed is 35 knots.
To verify this, use the tailwind distance.

\[
\frac{3}{4}(p + w) = 210
\]

\[
\frac{3}{4}(7w + w) = 210
\]

\[
\frac{3}{4}(8w) = 210
\]

\[6w = 210\]

\[\frac{1}{6}6w = 210\left(\frac{1}{6}\right)\]

\[w = 35\]

To find the speed of the plane, use either distance from the chart.

\[p - w = 210\]

Substitute 35 for \(w\):

\[p - 35 = 210\]

\[p = 210 + 35\]

\[p = 245\]

The speed of the plane is 245 knots.

\[
\frac{3}{4}(p + w) = 210
\]

This can be verified by using the other distance similarly:

Substitute 35 for \(w\):

\[
\frac{3}{4}(p + 35) = 210
\]

\[4\left[\frac{3}{4}(p + 35)\right] = (210)4\]

\[3(p + 35) = 840\]

\[\frac{1}{3}[3(p + 35)] = (840)^{1/3}\]

\[p + 35 = 280\]

\[p = 280 - 35\]

\[p = 245\]

**Exercise 1:**

With a tailwind, a plane travels 227 nautical miles in 30 minutes. With a headwind, it travels 207 nautical miles in 30 minutes. Find the speed of the plane in still air.

**Exercise 2:**

With a headwind, a plane travels 270 nautical miles in one hour. With a tailwind, the plane travels 220 nautical miles in 40 minutes. Find the speed of the plane in still air.
Crosswinds

The **heading** of a plane is the direction in which the plane is pointed, while the **course** of a plane is the direction of its actual path, which is not necessarily the direction in which the plane is pointed.

Wind often causes a plane to **drift** from its heading or direction. To stay on course, the Wright brothers constantly adjusted the heading of their flying machine by pulling cables attached to flaps and the flexible wings.

Pilots must calculate the effect the wind will have on the plane so that they can set the automatic navigation system at the correct heading. The diagram illustrates the effect of a crosswind, a wind blowing against the side of the plane.

In still air, a plane would travel due east along $\overline{AC}$.

With a crosswind blowing in the direction of $\overline{AB}$, the plane actually travels in the direction of $\overline{AD}$. $\overline{AD}$ is called the course of the plane.

$\angle CAD$ is called the drift angle.

**Example:**

Find the course, the speed in the wind, and the drift angle of a plane headed at $130^\circ$ when flying at 240 knots in still air if there is a crosswind of 30 knots blowing from direction $40^\circ$.

**Solution:**

Draw a diagram of the situation.

$\angle AXC$ is the drift angle.

$\angle AXB = 40^\circ + 180^\circ - 130^\circ = 90^\circ$

Using alternate interior angles:

$\angle CAX = \angle AXD$

$\angle AXD = 180^\circ - \angle AXB = 180^\circ - 90^\circ = 90^\circ$

$\angle CAX = 90^\circ$
Using ΔXAC and the Law of Cosines

\[ c^2 = a^2 + b^2 - 2ab \cos \theta, \]
\[ XC^2 = AC^2 + XA^2 - 2AC \cdot XA \cos A \]
\[ XC^2 = 30^2 + 240^2 - (2 \cdot 30) \cdot 240 \cos 90^\circ \]
\[ XC^2 = 900 + 57,600 - 0 = 58,500 \]
\[ XC = 241.9 \]

or, using the Law of Sines:

\[ \frac{\sin \angle AXC}{AC} = \frac{\sin \angle CAX}{XC} \]
\[ \frac{\sin \angle AXC}{30} = \frac{\sin 90^\circ}{241.9} = \frac{1}{241.9} \]
\[ \sin \angle AXC = \frac{30 \cdot 1}{241.9} = 0.1240 \]
\[ \angle AXC = 7.1^\circ \]

The course of the plane is 130° + 7.1° = 137.1°.
The plane is traveling at 241.9 knots with a drift angle of 7.1° and a course of 137.1°.

**Exercise 1:**

A pilot is to fly on course 72° in a wind blowing 40 knots from direction 134°. If the plane’s speed is 220 knots in what direction must he head the plane and what will be the speed of the plane in the wind?

**Exercise 2:**

A plane is headed in direction 120° with a speed of 180 knots in still air. The course is 105° with a speed in the wind of 160 knots. Find the speed and direction from which the wind is blowing.

Round answers to the nearest tenth.
Born of Dreams Inspired by Freedom!